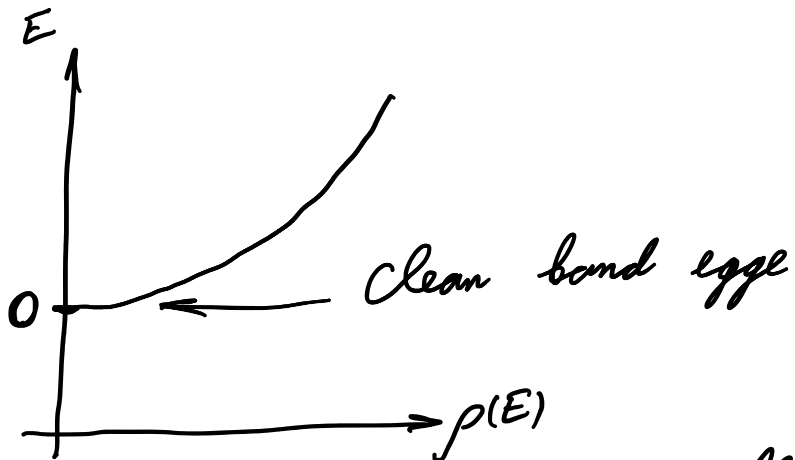
Lifshitz tails. States in the forbidden band.

$$\left(-\frac{\hbar^2 \nabla}{2m} + U(r)\right)\Psi = E\Psi$$

Assume, $U(r)$ has Gaussian distribution
It may form then arbitrarily deep potential wells




If potential forms a well of width $L$
and depth $|E| + \frac{m\hbar^2}{2L^2}$


The probability of such a fluctuation in volume $L^d$ is

$$-\frac{1}{\varkappa}\left(|E| + \frac{\hbar^2}{2mL^2}\right)^2 L^d$$

$\sim e$

$\varkappa$ - disorder strength

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One may show that for disorder with

One may show that for disorder with the correlation function

$$\langle U(\vec{r})\, U(\vec{r}\,')\rangle = \varkappa\, \delta(\vec{r}-\vec{r}\,')$$

The average disorder potential

$W = \frac{1}{V}\int_\Omega U(\vec{r})\, d\vec{r}$ has the distribution

with $P_\Omega(W) = \sqrt{\frac{2\pi V}{\varkappa}}\, e^{-\frac{W^2 V}{2\varkappa}}$

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Minimise the exponent

$$\left(|E| + \frac{\hbar^2}{2mL^2}\right)^2 L^d \longrightarrow \min$$

$$d\,L^{d-1}\left(|E| + \frac{\hbar^2}{2mL^2}\right)^{\cancel{2}} + 2\,\frac{-2\hbar^2}{2L^3 m}\cancel{\left(|E| + \frac{\hbar^2}{2mL^2}\right)} L^d = 0$$

$$|E| + \frac{\hbar^2}{2mL^2} = \frac{2}{d}\frac{\hbar^2}{mL^2} \longrightarrow$$

$$\longrightarrow \frac{\hbar^2}{2mL^2} = \frac{|E|\,d}{4-d}$$

$$L = \left(\frac{4-d}{2d}\frac{\hbar^2}{m|E|}\right)^{\frac{1}{2}}$$

The exponential:

$$\frac{1}{\varkappa}\left(\frac{4}{4-d}|E|\right)^2\left(\frac{4-d}{2d}\right)^{\frac{d}{2}}\left(\frac{\hbar^2}{m|E|}\right)^{\frac{d}{2}} =$$

$\propto (4-d \quad ) \quad (2d) \quad (m|E|)$

$$= \frac{16}{x} \frac{(4-d)^{\frac{d}{2}-2}}{(2d)^{\frac{d}{2}}} (\hbar^2/m)^{\frac{d}{2}} |E|^{2-\frac{d}{2}}$$

The DoS $\boxed{\rho(E) \propto e^{-C|E|^{2-\frac{d}{2}}}}$

– Lifshitz tail

In semiconductors in general there is a finite density of states at all energies